Binary, Hexadecimal Conversions Lesson

Read, Discuss and Answer part 1_{bin}

Along time ago, in a land far away, some really smart people figured out a great new notation for representing 10-based numbers, the Hindu-Arabic numerals. The Europeans of the times used Roman numerals and thought this “new math” was scary so they strongly resisted it for several centuries. However, the Hindu-Arabic number system was not to be defeated and is what many countries use today. The reason this is important is that you need to realize two things: 1) Hindu-Arabic numerals are really a shorthand notation for base 10 exponential expressions. 2) The base of ten (sometimes abreviated to ‘dec’) is not the only, or even the best, base for a number system.

Because you have been trained with Arabic numbers since birth, when you see the number 159, you have no problem conceptualizing how “many” that represents. Look at 159 as its actual exponential expression and you can better appreciate what the Hindu-Arabic numerals do for you.

```
1(10^2) + 5(10^1) + 9(10^0) = 159 dec
```

1) With your partner quickly copy the following table on paper, then discuss and fill in the empty boxes by converting between 10-based Arabic numbers and their exponential expressions:

5(10^2) + 2(10^1) + 1(10^0)	= 521
3(10^1) + 2(10^0)	=
	= 107
	= 861
4(10^2) + 0(10^1) + 2(10^0)	=
6(10^3) + 8(10^2) + 2(10^1) + 9(10^0)	=
	= 7957

Read, Discuss and Answer part 10_{bin}

Now a little deeper challenge. There is nothing naturally special about ten. Humans found agreement about a base 10 (or dec) number system because we have 10 fingers and 10 toes. If a really big meteor had not hit the earth 65 million years ago and the dinosaurs were writing this, they would likely use a 6-based number system, and it would work just fine.

2) Computers, at their heart-of-hearts, are a collection of switches. Switches have two positions or digits, on and off. We represent these with the digits 1 and 0 respectively. A computer’s favorite number system is base 2 and is called binary (sometimes abreviated to ‘bin’). As before, quickly copy the following table on paper, study the examples with your partner and fill in the missing numbers and expressions:

Exponential Expression	Bin	Dec
1(2^2) + 1(2^1) + 0(2^0)	= 110	= 6
1(2^3) + 0(2^2) + 1(2^1) + 1(2^0)	=	= 11
	= 1100	= 12
	=	= 9
1(2^4) + 0(2^3) + 1(2^2) + 1(2^1) + 1(2^0)	=	=
	= 11010	=
	=	= 25

Read, Discuss and Answer part 11_{bin}

Because binary numbers can be cumbersome for humans to work with, progammers created a base 16 number system called hexadecimal. Hexadecimals are very efficiently used by computers because 16 is a power of 2 and hexadecimals also allow very large numbers to be shown with many fewer digits than binary or base 10 systems.

For example: `7b3f`(hex) = `31551`(dec) = `111101100111111`(bin) .

Notice the letters that serve as digits within some hexadecimal numbers. Do not get scared like the Europeans who loved their Roman numerals! These letters are necessary because just as an efficient 10 base number system needs 10 symbols available for each digit, a hexadecimal number needs 16 symbols. This exceeds what the Arabs and Hindus gave us so computer scientists substituted letters for the missing symbols:

```
dec:  0  1  2  3  4  5  6  7  8  9        (ten symbols including zero)
hex:  0  1  2  3  4  5  6  7  8  9  a  b  c  d  e  f   (16 symbols)
                                    10 11 12 13 14 15
```

3) Study the examples with your partner and fill in the missing numbers and expressions:

Exponential Expression	Hex	Dec
5(16^1) + 11(16^0)	= 5b	= 91
1(16^1) + 13(16^0)	=	= 29
	= e3	= 227
	=	= 127
11(16^2) + 2(16^1) + 5(16^0)	=	=
	= a12	=

When you are finished, check your answers with your teacher and be ready to justify them. You will then have an opportunity to complete additional hexadecimal and binary conversions with your partner.